

Abstract Interpretation with Frama-C

EJCP 2021

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long ra
t for 0 =>
C1) if (m
tmp2
e of the

tmp2[0] = 1 << (n0 - 1) else if (tmp1[0]) >> 1 << (n0 - 1) tmp2[0] = 1 << (n0 - 1) else tmp2[0] = tmp1[0]; /* Then the second part takes the first one...
tmp1[0] = 0; k = 0; k <= 5; k++) tmp1[0] += mc2[0][k] * tmp2[0][k]; /* The [k] coefficient of the matrix product MC2*TMP2, that is, *MC2*(TMP1) = MC2*(M1) = MC2*(M1)*MC1
l = 1; tmp1[0] >> 1; /* Final rounding: tmp2[0] is now represented on 9 bits. *if (tmp1[0] < -256) m2[0] = -256; else if (tmp1[0] > 255) m2[0] = 255; else m2[0] = tmp1[0];

- ▶ Software is more and more pervasive in embedded systems...
- ▶ ...and keeps getting larger
- ▶ Tests and code review too costly beyond a certain size and coverage criterion
- ▶ Need for **correct** tools
 - ✓ Detect all potential issues
 - ✗ May issue spurious warnings
 - ✗ Impossible for an automated tool to warn for all real issues and only for them (Rice theorem)

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Context

Abstract Domains

Termination

Improving precision

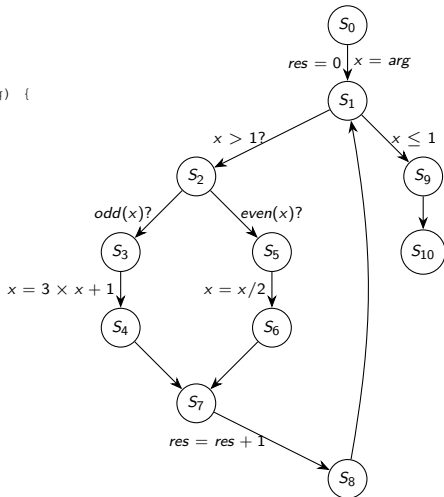
Setting Analysis Context

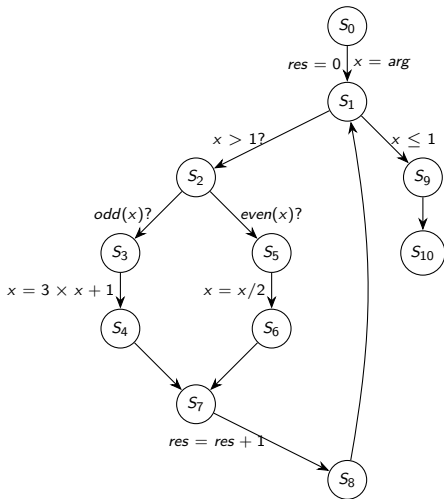
Control-Flow Graph

```

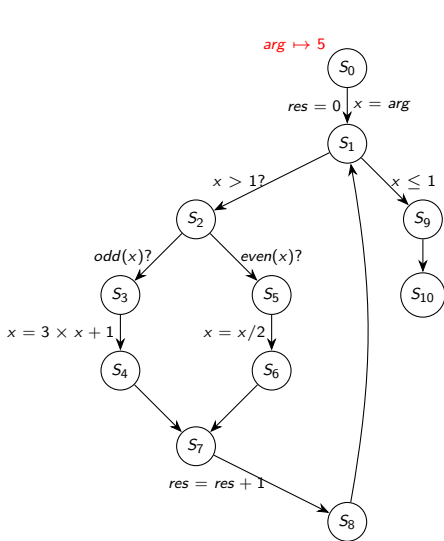
mpz_ptr syracuse(mpz_t res, const mpz_t arg) {
  mpz_t x;
  mpz_init_set_ui(res, 0UL);
  mpz_init_set(x, arg);
  while (mpz_cmp_ui(x, 1UL) > 0) {
    mpz_out_str(stdout, 10, x);
    putchar('\n');
    if (mpz_odd_p(x)) {
      mpz_mul_ui(x, x, 3UL);
      mpz_add_ui(x, x, 1UL);
    } else {
      mpz_cdiv_q_ui(x, x, 2UL);
    }
    mpz_add_ui(res, res, 1UL);
  }
  mpz_clear(x);
  return res;
}

```



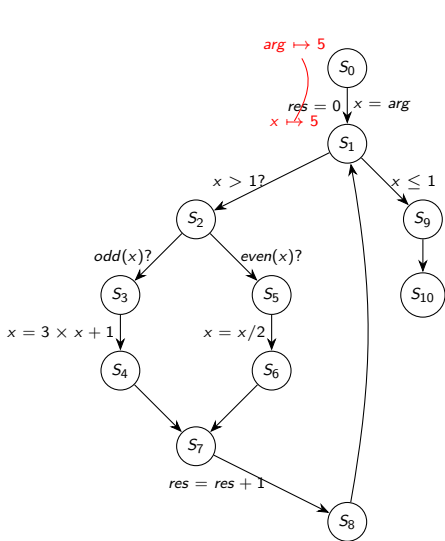


- ▶ Initial state on start node
- ▶ Transfer functions across edges
- ▶ Collecting semantics: recall all states associated to each program point

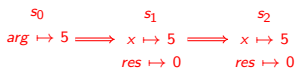
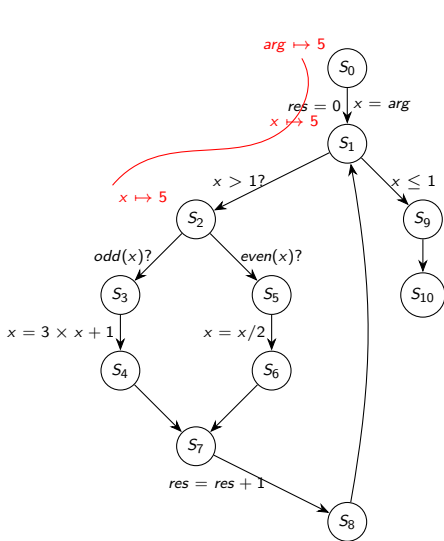


s_0
 $arg \mapsto 5$

- ▶ Initial state on start node
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$$\begin{array}{l} s_0 \\ arg \mapsto 5 \end{array} \Longrightarrow \begin{array}{l} s_1 \\ x \mapsto 5 \\ res \mapsto 0 \end{array}$$

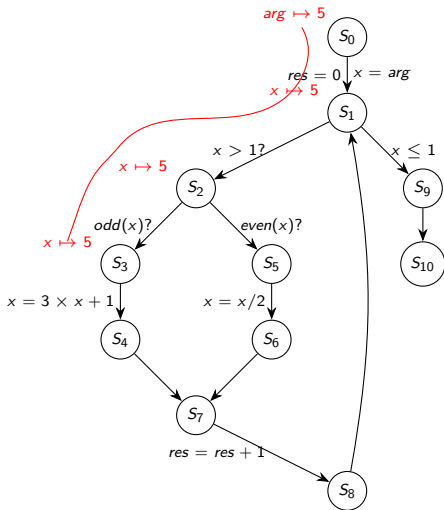
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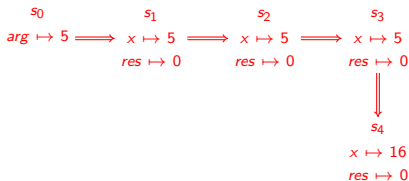
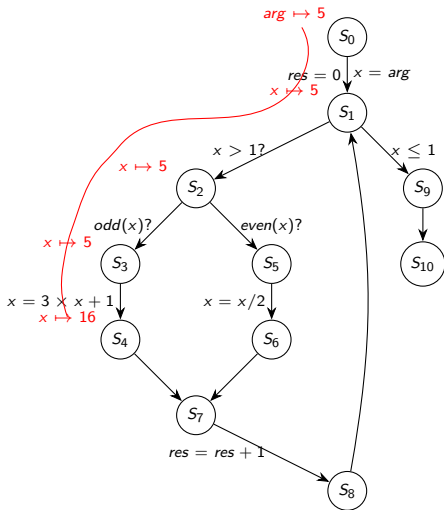
Trace Semantics

$$\begin{array}{cccc}
 s_0 & & s_1 & & s_2 & & s_3 \\
 \text{arg} \mapsto 5 & \Longrightarrow & x \mapsto 5 & \Longrightarrow & x \mapsto 5 & \Longrightarrow & x \mapsto 5 \\
 & & \text{res} \mapsto 0 & & \text{res} \mapsto 0 & & \text{res} \mapsto 0
 \end{array}$$

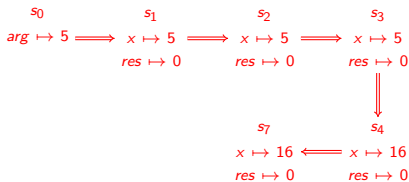
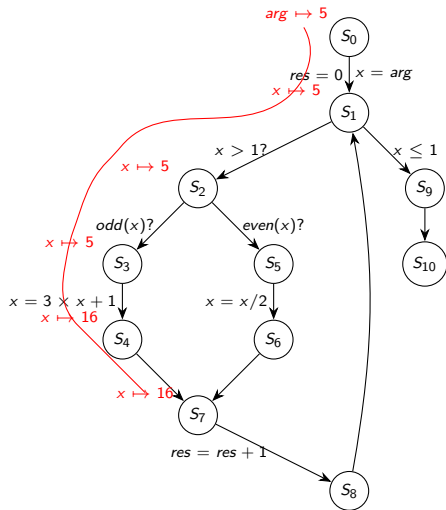


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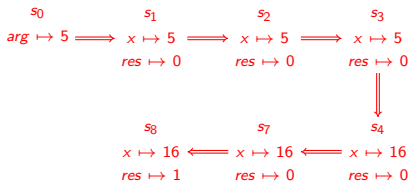
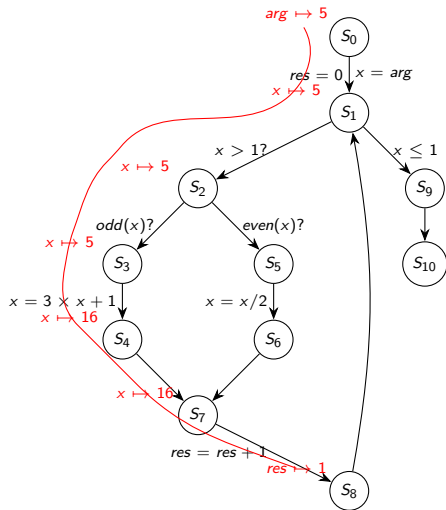


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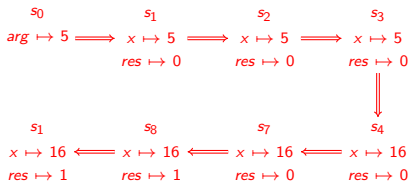
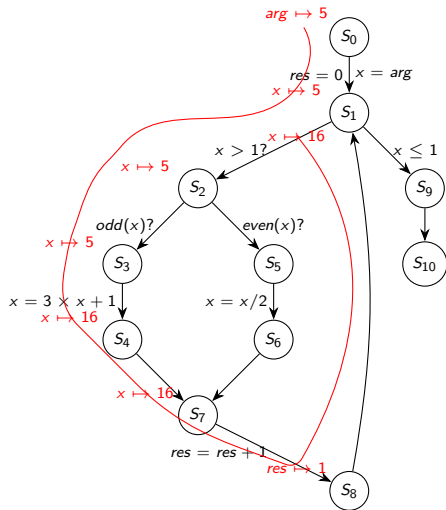
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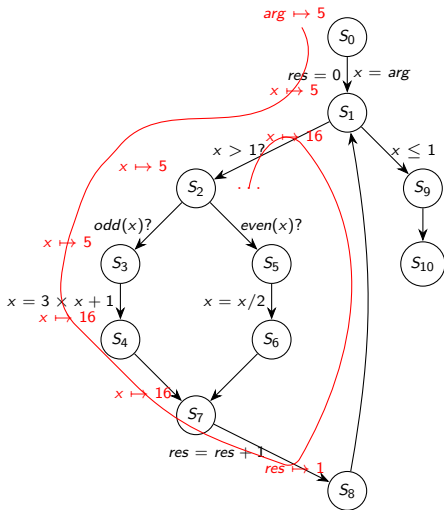


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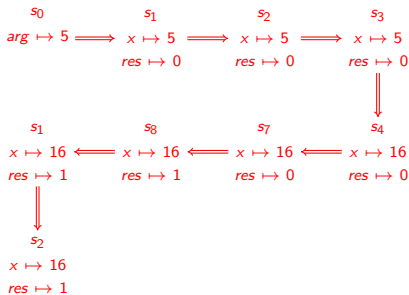
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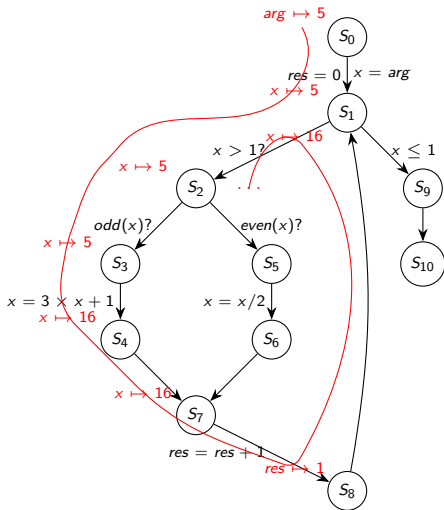
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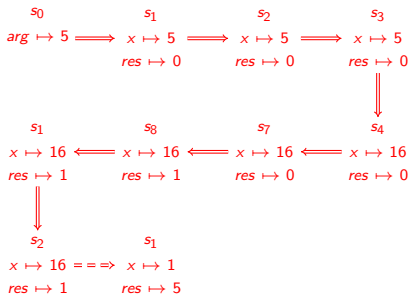
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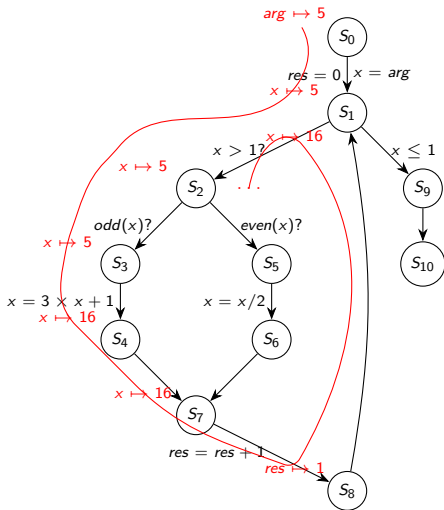
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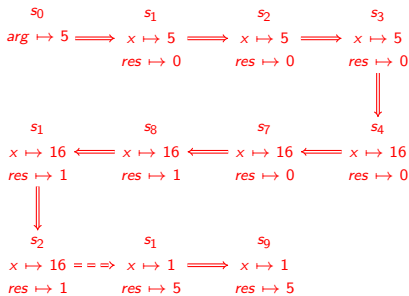
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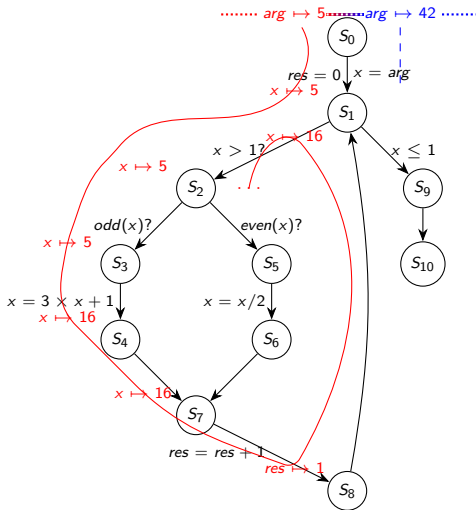
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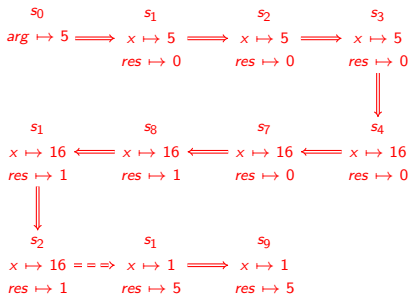
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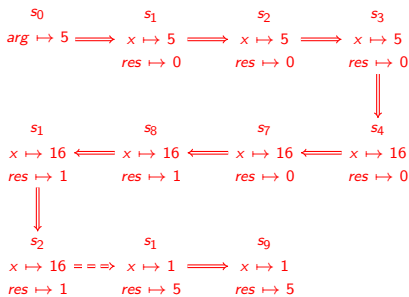
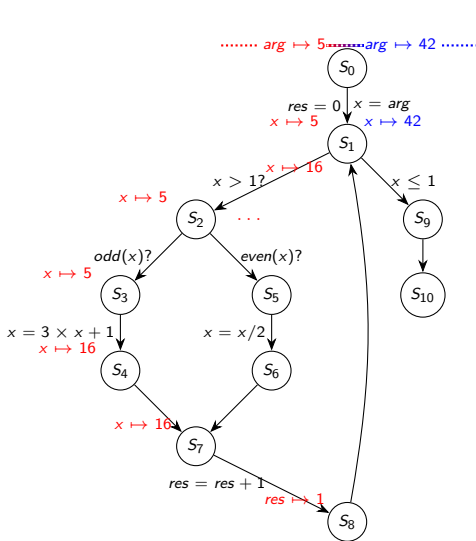


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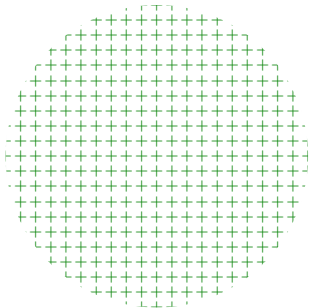
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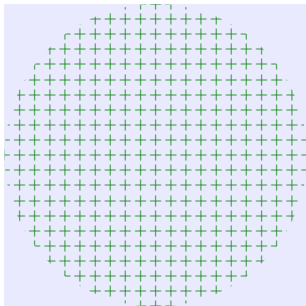
Static Analysis framework

- ▶ Replace set of states ...
- ▶ ... by one element in an abstract lattice
- ▶ Correctness: do not miss any possible concrete state
- ▶ Over-approximation and false alarms
- ▶ Termination: ensure analysis always terminates
- ▶ Trade-off between precision and computation time
- ▶ Abstract interpretation: systematic way to build correct and terminating analyses (Galois connexions and widening)

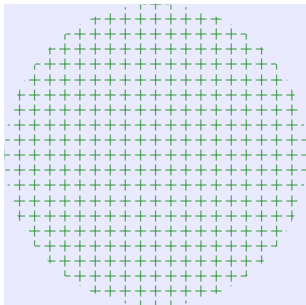


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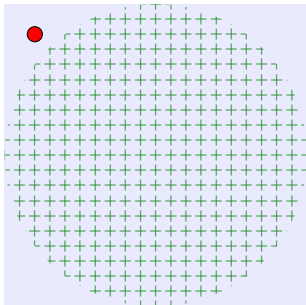


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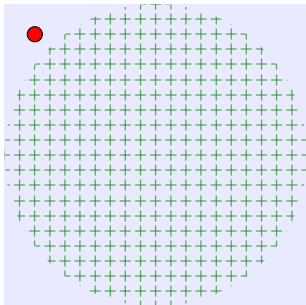
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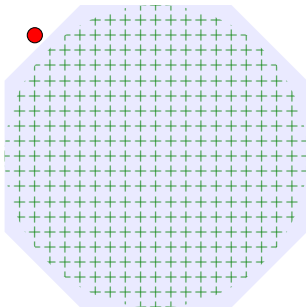
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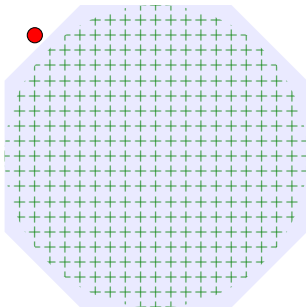
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A few tools

- ▶ **Polyspace Verifier**: check absence of runtime errors (C/C++/Ada)

<https://fr.mathworks.com/products/polyspace.html>

- ▶ **ASTRÉE**: absence of runtime errors **without false alarm** in SCADÉ-generated code

<https://www.absint.com/astree/index.htm>

- ▶ **Verasco**: certified (in Coq) analyzer

<http://compcert.inria.fr/verasco/>

- ▶ **aiT/StackAnalyzer**: WCET and stack size (assembly code)

<https://www.absint.com/ait/>

- ▶ **FLUCTUAT**: accuracy of floating-point computations and origin of rounding errors

<https://www.lix.polytechnique.fr/~putot/fluctuat.html>

- ▶ **Frama-C**: platform for analyzing C code, including through abstract interpretation

<https://frama-c.com>

- ▶ **MoPSA**: modular platform for abstract interpretation

<https://gitlab.com/mopsa/mopsa-analyzer>

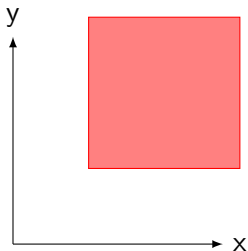
- ▶ A Framework for modular analysis of C code.
- ▶ <https://frama-c.com/>
- ▶ Developed at CEA Tech List and Inria
- ▶ Released under LGPL license (v23.0-rc1 Vanadium in June 2021)
- ▶ Kernel based on CIL (Necula et al. – Berkeley).
- ▶ ACSL annotation language.
- ▶ Extensible platform
 - ▶ Collaboration of analyses over same code
 - ▶ Inter plug-in communication through ACSL formulas.
 - ▶ Adding specialized plug-in is easy

Relational and Non-relational Lattices

Non-relational domain

- ▶ Considers each variable independently
- ✓ Simpler and less costly
- ✗ lose properties over 2+ variables

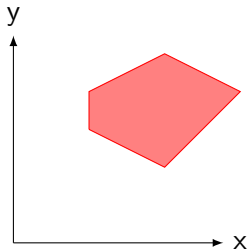
Example: intervals



Relational domain

- ▶ Considers several variables at once
- ✓ More precise
- ✗ More complex and costly

Example: Polyhedra



Corresponding Abstract Domain

- small set of integers (by default, cardinal ≤ 8)
- \uplus integer interval

Examples

- ▶ $\{0; 40; \}$ = 0 or 40
- ▶ $[0..40]$ = an integer between 0 and 40 (inclusive)
- ▶ $[-..-]$ = any integer (within the bound of the corresponding integral type)

Question

if x is in the interval $[-10 .. 10]$ before the execution of statement

```
if (x==0) { y = 14; }
else { y = x<0 ? 13 : x + 2; }
```

What is the value associated to y after the statement?

Answers

- a $[-8 .. 14]$
- b $[2 .. 13]$
- c $[2 .. 14]$
- d $[3 .. 14]$

- ▶ Combining abstract domains
- ▶ **reduce** abstract value from one domain using information from the other
- ✗ In practice, not as simple and generic as it looks
- ✗ Combining transfer function is complex

Question

We have information from two domains:

Intervals:

▶ $x \in [0; 20]$

▶ $y \in [5; 10]$

Octagons:

$$0 \leq x - y \leq 20$$

What can be said about x and y ?

Answers

▶ a $x \in [0; 20], y \in [5; 10]; 0 \leq x - y \leq 20$

▶ b $x \in [5; 20], y \in [5; 10], 0 \leq x - y \leq 15$

▶ c $x \in [5; 20], y \in [5; 10], 0 \leq x - y \leq 10$

▶ d $x \in [5; 20], y \in [0; 20], 0 \leq x - y \leq 20$

Base Address

- Global variable
- ⊕ Formal parameter of main function
- ⊕ literal string constant
- ⊕ NULL
- ⊕ ...

Addresses

- ▶ Base address + Offset (integer)
- ▶ Integer representation: product of interval and modulo (aka alignment) information
- ▶ Each base has a maximal valid offset
- ▶ Abstract Values are sets of addresses

Abstract Domain

valid left value

address

- × initialized?
- × *not dangling pointer?*

Example

```

int x, y;
if (e) x = 2;
L: if (e) y = x + 1;
  
```

- ▶ At L , we know that x equals 2 iff it has been initialized
- ▶ Depending on the complexity of e , we know that y equals 3 if x equals 2

Question

if a is an array of size 3, initialized to 0, and c in $[0 \dots 2]$ what would be the content of a after executing the following statement:

```
if (c) { a[c] = c; } else a[1] =3;
```

Answers

- a `a[0] IN {0}, a[1] IN {0,1,3}, a[2] IN {0,2}`
- b `a[i] IN {0,1,2,3} for all indices`
- c `a[0] IN {0}, a[1] IN {0,1,2,3}, a[2] IN {0,1,2}`
- d `a[0] IN {0}, a[1] IN {1,3}, a[2] IN {2}`

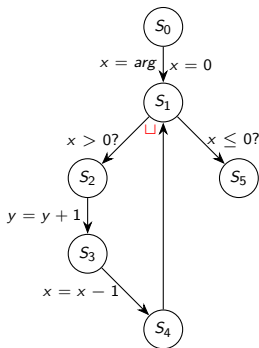
- ▶ New domains can provide additional information:
 - ▶ equalities between values
 - ▶ values of symbolic locations
 - ▶ gauges, affine relation wrt number of loop steps
- ▶ Possible to add new domains
- ▶ Inter-domain communication done through **queries**:

```

val extract_expr :
  (exp -> value evaluated) ->
  state -> exp -> (value * origin) evaluated
  
```

```

val extract_lval :
  (exp -> value evaluated) ->
  state -> lval -> typ -> location ->
  (value * origin) evaluated
  
```

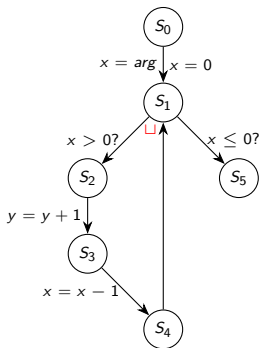


$$S_1 \text{ (before)} \quad S_4 \quad = \quad S_1 \text{ (after)}$$

$$y \in [0; 0] \sqcup [1; 1] = [0; 1]$$

- ▶ for loop nodes, state grows slowly at each step
- ▶ convergence could require infinite time
- ▶ replace \sqcup with widening operator ∇ :

correctness $x \sqcup y \sqsubseteq x \nabla y$
 termination no infinitely growing sequence
 $x_0 \nabla x_1 \nabla \dots \nabla x_n \dots$

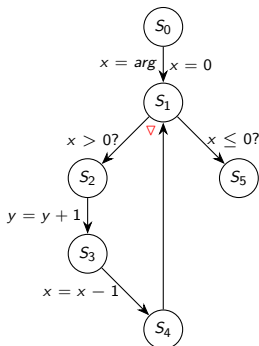


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$$y \in [0; 2] \sqcup [1; 3] = [0; 3]$$

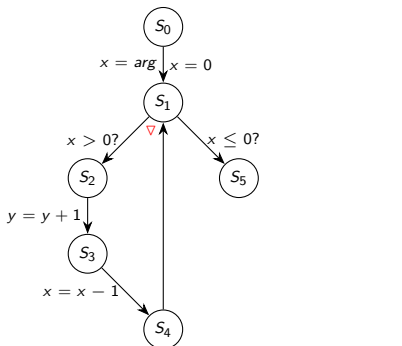
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S_1 (before) S_4 S_1 (after)
 $y \in [0; 2]$ ∇ $[1; 3]$ $= [0; +\infty]$

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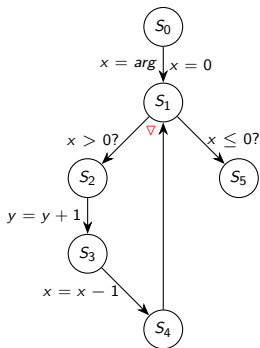


$$S_1 \text{ (before)} \quad S_4 \quad S_1 \text{ (after)} \\
 y \in [0; 2] \quad \nabla \quad [1; 3] = [0; +\infty]$$

lower bound stable:
 don't change

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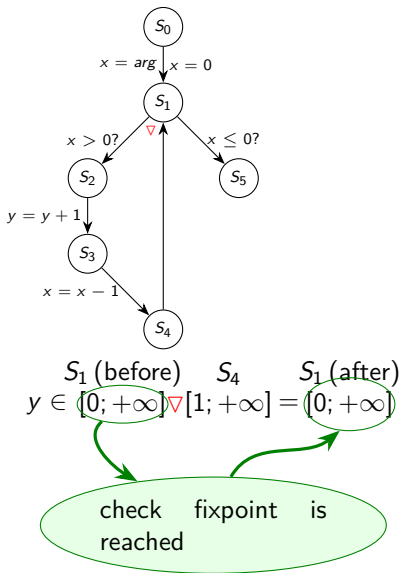
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upper bound grows:
widen interval



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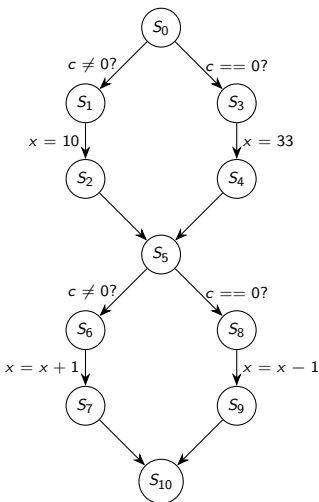
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Widening

- ▶ Controlling widening: `-eva-widening-delay`,
`-eva-widening-period`
- ▶ Controlling widening bounds:
`widen_hints v, 10, 100, 1000;`

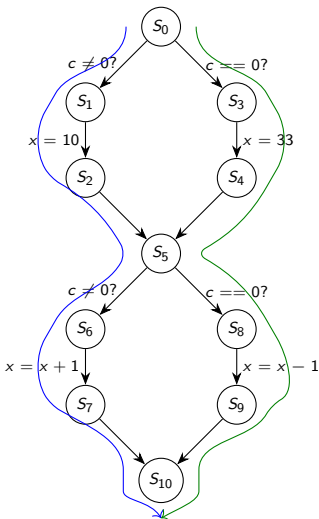
Loop unrolling

- ▶ `-eva-auto-loop-unroll n`
- ▶ `-eva-min-loop-unroll n`
- ▶ or with annotations on specific loops



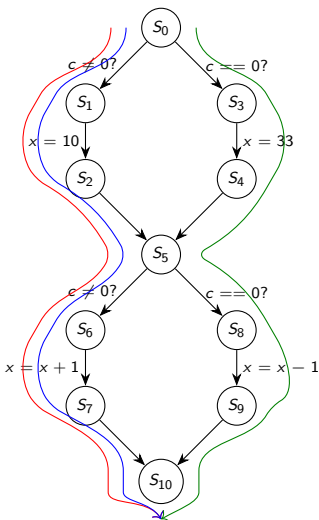
	S_5	S_6	S_8	S_{10}
c	\mathbb{Z}	\mathbb{Z}	0	\mathbb{Z}
x	[10; 33]	[10; 33]	[10; 33]	[9; 34]

- ▶ Consider several abstract traces separately...
- ▶ ...At least for some time
- ✓ More precise than collecting semantics
- ✗ Finding appropriate partition is difficult



	S_5	S_6	S_8	S_{10}
c	0	\perp	0	0
x	33	\perp	33	32
c	\mathbb{Z}	\mathbb{Z}	0	\mathbb{Z}
x	10	10	10	[9; 11]

- ▶ Consider several abstract traces separately...
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	S_5	S_6	S_8	S_{10}
c	0	\perp	0	0
x	33	\perp	33	32
c	$[1; +\infty]$	$[1; +\infty]$	\perp	$[1; +\infty]$
x	10	10	\perp	11
c	$[-\infty; -1]$	$[-\infty; -1]$	\perp	$[-\infty; -1]$
x	10	10	\perp	11

- ▶ Consider several abstract traces separately...
- ▶ ...At least for some time
- ✓ More precise than collecting semantics
- ✗ Finding appropriate partition is difficult

Slevel options

- ▶ option `-eva-slevel`: allows Eva to explore n separated paths before joining them
- ▶ option `-eva-slevel-function`: same as previous, but for a particular function
- ▶ Can also use ACSL annotations to partition the state

ACSL Annotations

use a disjunction (that covers all possible cases) to split the state, and appropriate `-eva-slevel`

```
/*@ assert A || B || C; */
```

Explicit split

```
/*@ split expr; */
```

...

```
/*@ merge expr; */
```

`expr` must evaluate to a small set of values (e.g. be a boolean expression)

- ▶ Which part of the code should be analyzed?
- ▶ `-main f` starts the analysis at function `f`
- ▶ `-lib-entry` indicates that the the initial global context is **not** 0-initialized
- ▶ `-eva-context-width`, `-eva-context-depth`
- ▶ Use of a driver function with some builtins to provide non-determinism:

```
void f_wrapper() {  
    setup_analysis_context();  
    f(arg_1, arg_2);  
}
```

Provide an “implementation” for Eva

- ▶ Assumed to match the real implementation
- ▶ Write stub directly in C (aimed at ease of analysis, not performance)
- ▶ Provide an ACSL specification
- ▶ `-eva-use-spec f`
- ▶ Use an Eva built-in (`-eva-builtin`)
- ▶ `-eva-builtins-list`

CERT ARR30-C bad code sample

```
static int *table = NULL;
static size_t size = 0;

int insert_in_table(size_t pos, int value) {
    if (size < pos) {
        int *tmp;
        size = pos + 1;
        tmp = (int *)realloc(
            table, sizeof(*table) * size);
        if (tmp == NULL) {
            return -1;    /* Failure */
        }
        table = tmp;
    }
    table[pos] = value;
    return 0;
}
```

- ▶ D. Delmas and J. Souyris: *ASTRÉE: from Research to Industry*, SAS 2007
- ▶ TrustInSoft startup (created 2013): <https://trust-in-soft.com/>
- ▶ A. Ourghanlian: Evaluation of static analysis tools used to assess software important to nuclear power plant safety. In *Nuclear Engineering and Technology*, vol 47 issue 2, 2015.
- ▶ A. Ebalard &al.: Journey to a RTE-free X.509 parser
<https://www.sstic.org/2019/presentation/journey-to-a-rte-free-x509-parser/>
- ▶ Open-Source Case Studies:
<https://github.com/Frama-C/open-source-case-studies>
- ▶ A. Maroneze: Analysis of the Chrony NTP server.
<https://frama-c.com/2018/06/19/Analyzing-Chrony-with-Frama-C-Eva.html>

General

- ▶ Correnson &al. Frama-C User Manual (v22 - Titanium). November 2020
- ▶ Kirchner &al. Frama-C, a Software Analysis Perspective, vol 37 of Formal Aspects of Computing, March 2015.

Eva

- ▶ Cuoq &al. Frama-C's value analysis plug-in. November 2020
- ▶ Blazy &al. Structuring Abstract Interpreters through State and Value Abstractions. VMCAI, January 2017

Course

- ▶ Patrick Cousot, MIT 2005

<http://web.mit.edu/afs/athena.mit.edu/course/16/16.399/www/>

Books

- ▶ Hanne Nielson, Flemming Nielson, and Chris Hankin. *Principles of Program Analysis*. Springer 1999
- ▶ Neil Jones and Flemming Nielson, *Abstract Interpretation: a Semantics-Based Tool for Program Analysis*. In *Handbook of Logic in Computer Science, vol. 4*, Oxford University Press 1994

Founding Articles

- ▶ Patrick and Radhia Cousot, *Abstract Interpretation: a Unified Lattice Model for Static Analysis of Programs by Construction or Approximation of Fixpoints*. PoPL'77
- ▶ Patrick Cousot and Nicolas Halbwachs, *Automatic Discovery of Linear Restraints Among Variables of a Program*. PoPL'78
- ▶ Patrick and Radhia Cousot, *Systematic Design of Program Analysis Frameworks*. PoPL'79
- ▶ <http://www.di.ens.fr/~cousot/COUSOTpapers.shtml>

Solutions to Quizzes

Question

if x is in the interval $[-10 \dots 10]$ before the execution of statement

```

if (x==0) { y = 14; }
else { y = x<0 ? 13 : x + 2; }
    
```

What is the value associated to y after the statement?

Answers

- a $[-8 \dots 14]$ **X**
- b $[2 \dots 13]$
- c $[2 \dots 14]$
- d $[3 \dots 14]$

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Question

We have information from two domains:

Intervals:

▶ $x \in [0; 20]$

▶ $y \in [5; 10]$

Octagons:

$$0 \leq x - y \leq 20$$

What can be said about x and y ?

Answers

▶ a $x \in [0; 20], y \in [5; 10]; 0 \leq x - y \leq 20$ ❌

▶ b $x \in [5; 20], y \in [5; 10], 0 \leq x - y \leq 15$

▶ c $x \in [5; 20], y \in [5; 10], 0 \leq x - y \leq 10$

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Question

if a is an array of size 3, initialized to 0, and c in $[0 \dots 2]$ what would be the content of a after executing the following statement:

```
if (c) { a[c] = c; } else a[1] =3;
```

Answers

a $a[0] \in \{0\}$, $a[1] \in \{0,1,3\}$, $a[2] \in \{0,2\}$

X

b $a[i] \in \{0,1,2,3\}$ for all indices

c

$a[0] \in \{0\}$, $a[1] \in \{0,1,2,3\}$, $a[2] \in \{0,1,2\}$

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c

`a[0] IN {0}, a[1] IN {0,1,2,3}, a[2] IN {0,1,2}`

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